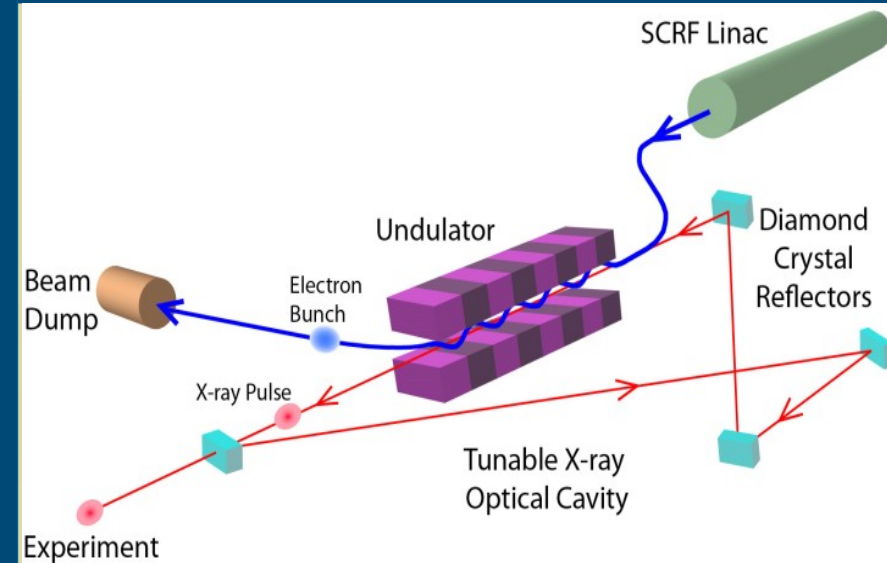


Beam dynamics for an x-ray free-electron oscillator (XFELO)



Ryan Lindberg

Accelerator Operations and Physics Group
Argonne National Laboratory

Coherence in particle and photon beams: past, present, and future
(Kwang-Je Kim Fest)
Friday March 15, 2019

Introduction

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- While SASE took off, the oscillator laid dormant for more than 20 years

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The XFEL revival paper^[2] could be used to outline my outline

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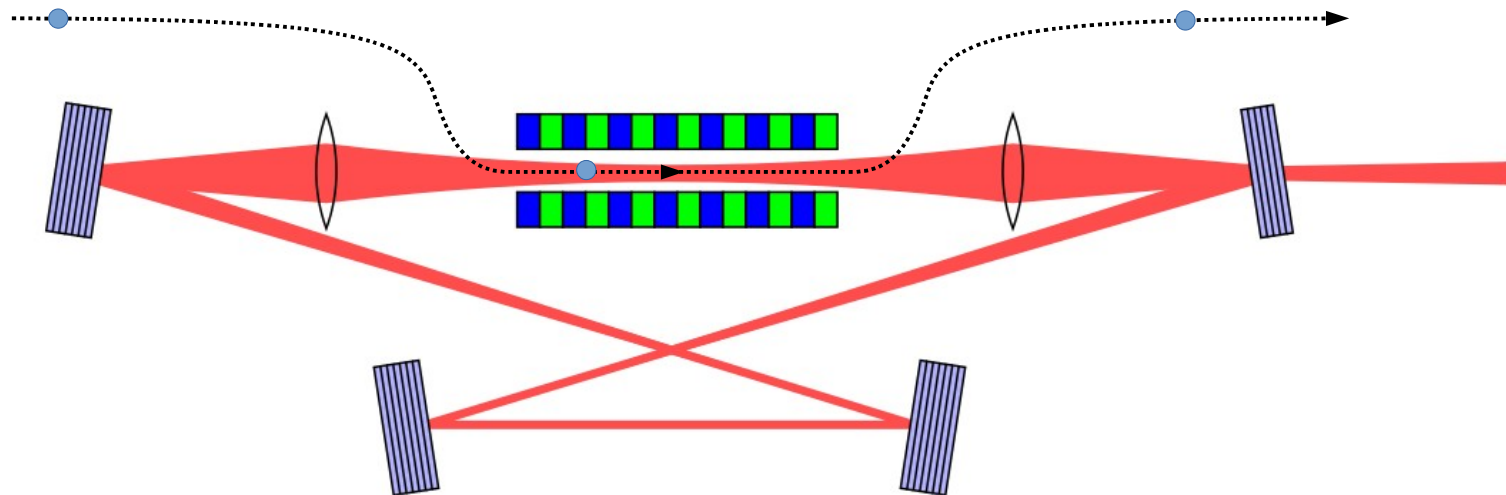
Recent investigations using superconducting linacs and storage rings

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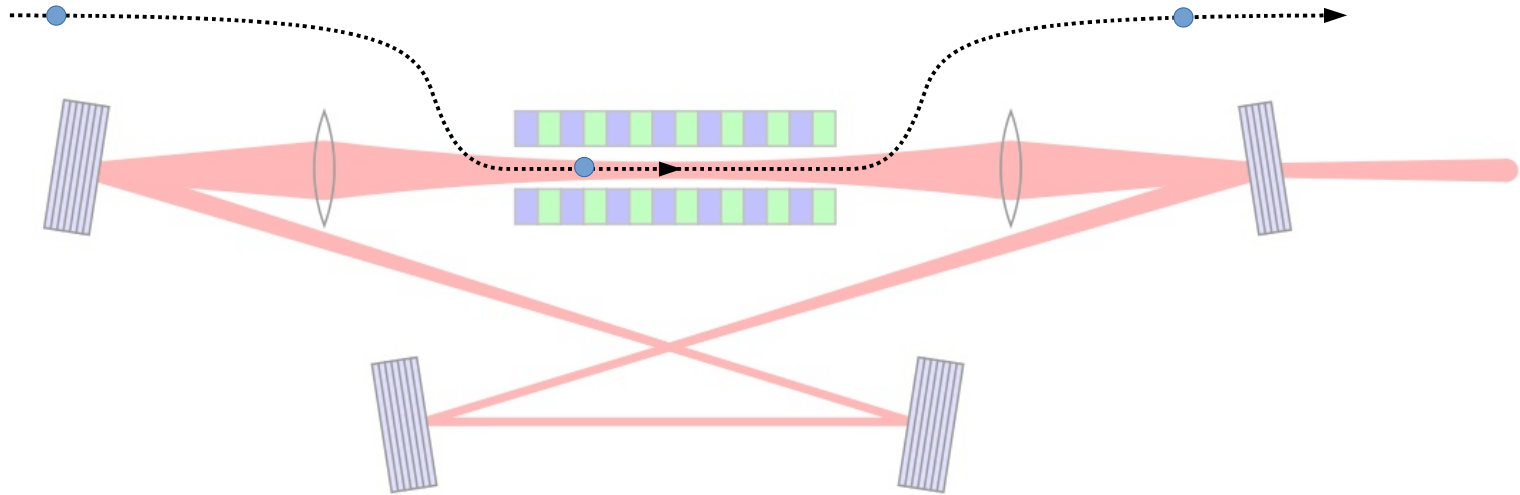
Some progress + future work

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X-ray FEL oscillator schematic



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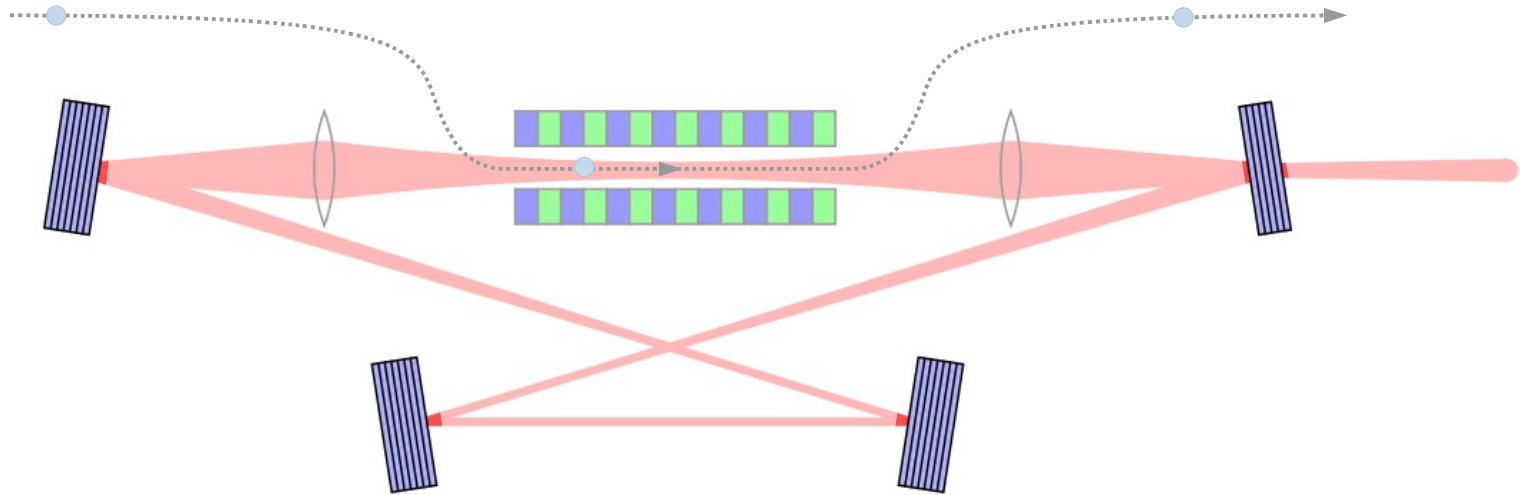


- High-brightness, low (10-300 A) peak current electron beam at high ($\sim$ MHz) repetition rate

The diagram illustrates a quantum communication setup. A central source, represented by two rows of blue and green blocks, emits particles (blue dots) along a horizontal path. The path is flanked by two lenses and four mirrors (blue rectangles). Red lines show the optical paths. A dashed line with an arrow indicates the direction of particle travel.

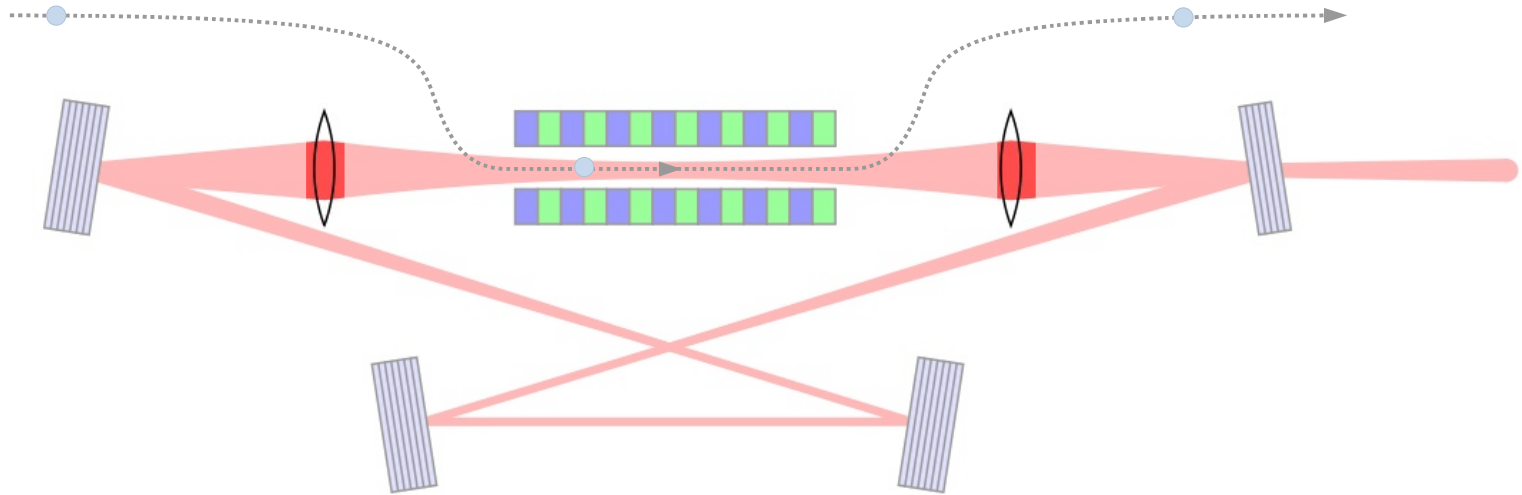
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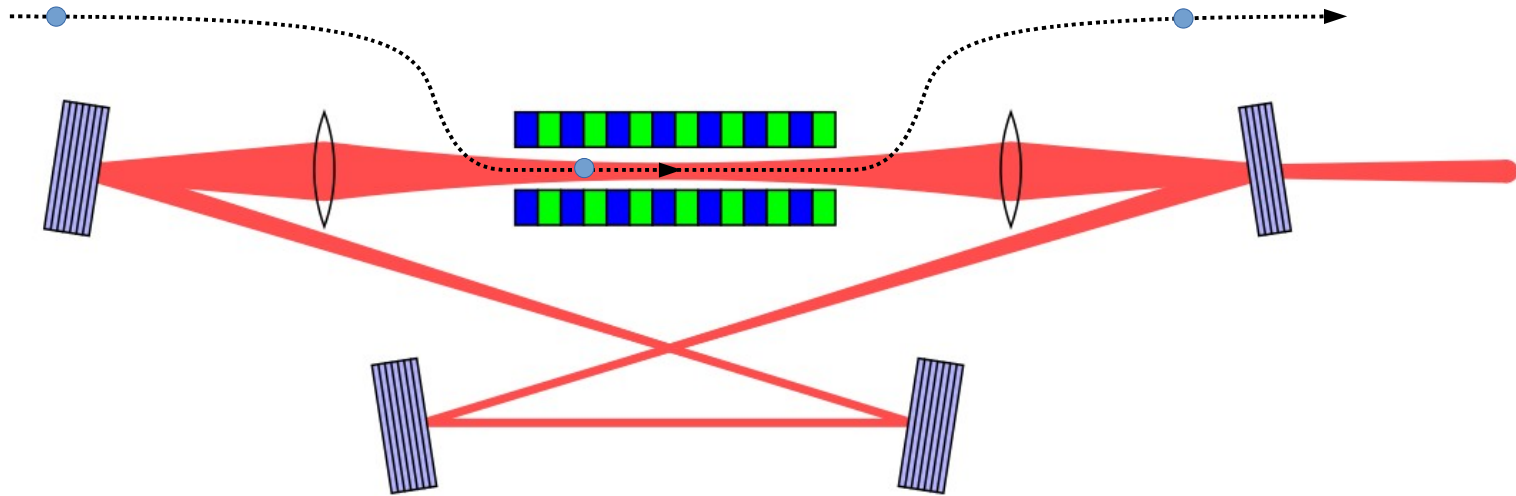
- High-brightness, low (10-300 A) peak current electron beam at high ($\sim$ MHz) repetition rate
- Undulator
- Low loss optical cavity
 - Perfect diamond crystals that reflect x-rays via Bragg diffraction [3 thick crystals with low loss, $(1-R) < 5\%$; 1 thin crystal to allow for $\sim 5\%$ transmission]

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 - Bow-tie shape is a wrapped-up monochromator that allows for wavelength tuning^[3,4]

[3] R.M.J. Cotterill, "A universal planar x-ray resonator" Appl. Phys. Lett. **12**, 403 (1968).

[4] K.-J. Kim and Y. Shvyd'ko. "Tunable optical cavity for an x-ray free-electron-laser oscillator," PRST-AB **12**, 030703 (2009).

XFELO is a complementary source to SASE

Characteristic	SASE	XFELO
Pulse duration	1 to 200 fs	200 to 2000 fs
Photons/pulse	$\sim 10^{12}$	$\sim 10^9$
Energy BW	~ 10 eV	$\sim 10^{-2}$ eV
Coherence	Transverse	Fully
Repetition rate	Variable	$\sim$ MHz
Stability	1-100% depending on chosen BW	$< 1\%$
Brightness	$\sim 10^{32}$	$\sim 10^{32}$

[XFELO Science](#)

1. Inelastic x-ray scattering
2. Nuclear resonant scattering
3. X-ray photo-emission spectroscopy
4. Hard x-ray imaging
5. X-ray photon correlation spectroscopy
6. ???

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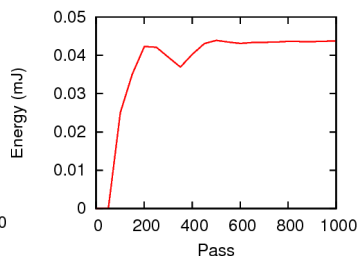
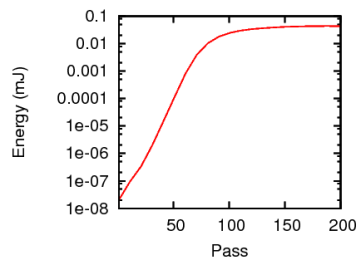
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Example^[5]

Beam energy γmc^2	7 GeV
Energy spread σ_γ/γ	2×10^{-4}
Norm. emittance ϵ_n	0.2 mm*mrad
Peak current I	10 A
Undulator periods N_u	3000
Undulator length L_u	53 m



[5] R.R. Lindberg, K.-J. Kim, Yu. Shvyd'ko, and W.M. Fawley. "Performance of the x-ray free-electron laser oscillator with crystal cavity" PRST-AB **14**, 010701 (2011).

XFELO is a complementary source to SASE

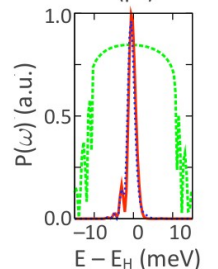
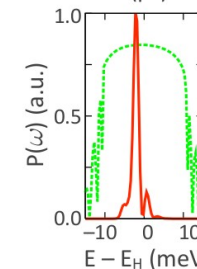
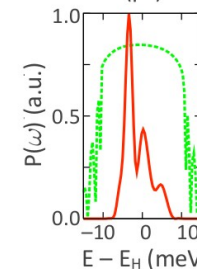
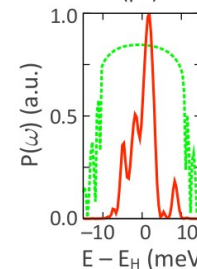
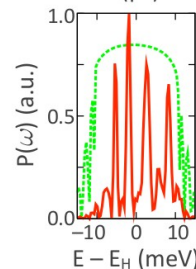
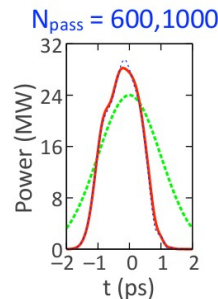
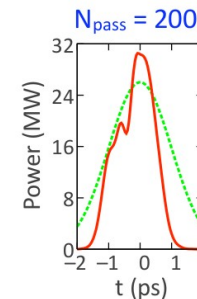
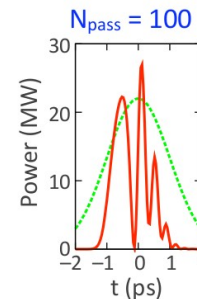
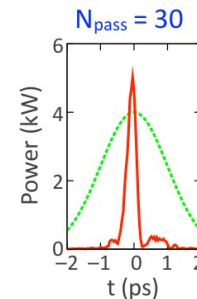
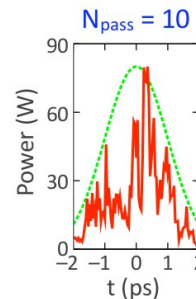
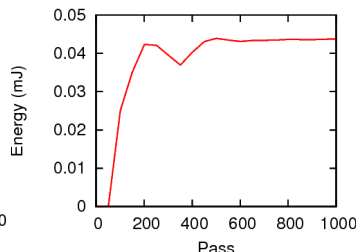
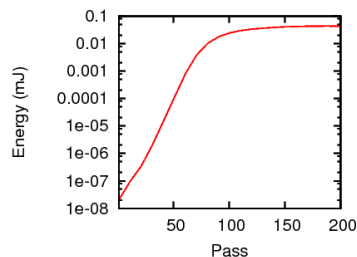
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$$\frac{\partial}{\partial n} E(t, n) = E(t, n) + \frac{G}{2} \left(1 - \frac{t^2}{2\sigma_e^2} \right) E(t, n) - \left(\frac{R}{2} - \frac{1}{\sigma_\omega^2} \frac{\partial^2}{\partial t^2} \right) E(t, n) + \ell \frac{\partial}{\partial t} E(t, n)$$

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➡ General solution is spanned by a sum of Gauss-Hermite modes

$$E_m(t, n) = e^{\Lambda_m n} e^{-t^2 \sigma_\omega^2 \ell / 2} \exp \left(-\frac{\sqrt{G} \sigma_\omega}{2\sigma_e} t^2 \right) H_m \left(G^{1/4} \sqrt{\frac{\sigma_\omega}{\sigma_e}} t \right)$$

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$$\Lambda_m = \frac{1}{2} \left[G - R - \frac{\sqrt{G}}{\sigma_e \sigma_\omega} (2m + 1) - \frac{\sigma_\omega^2 \ell^2}{2} \right]$$

Gain is reduced when electron beam duration σ_e approaches the inverse bandwidth of crystal $1/\sigma_\omega$

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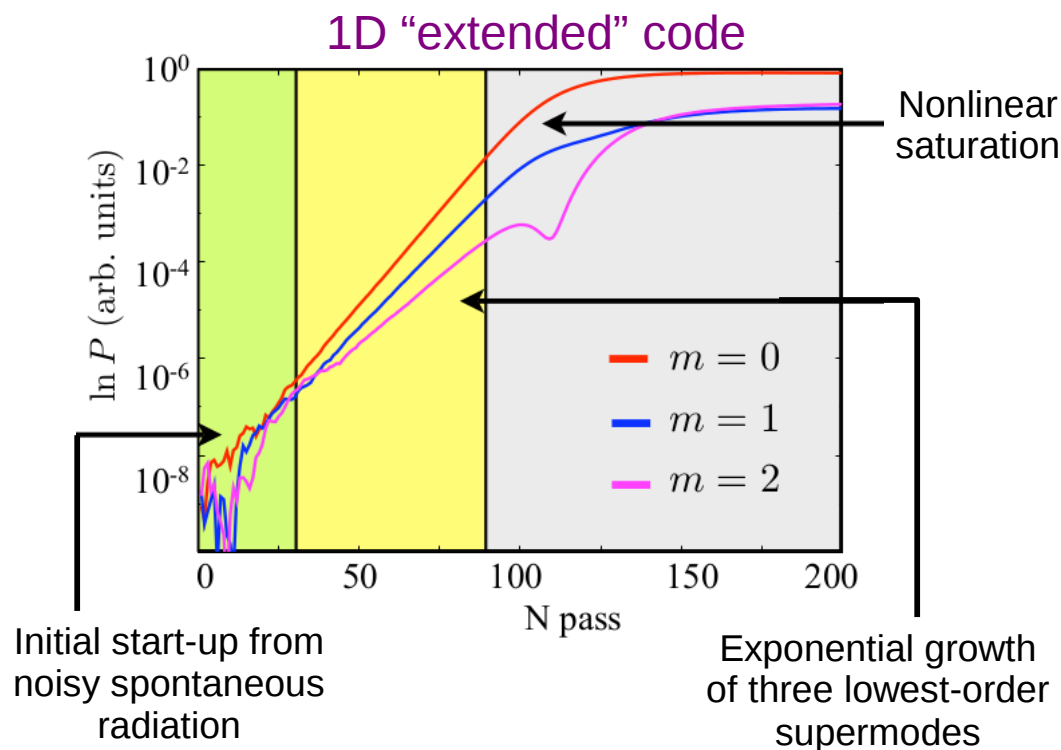
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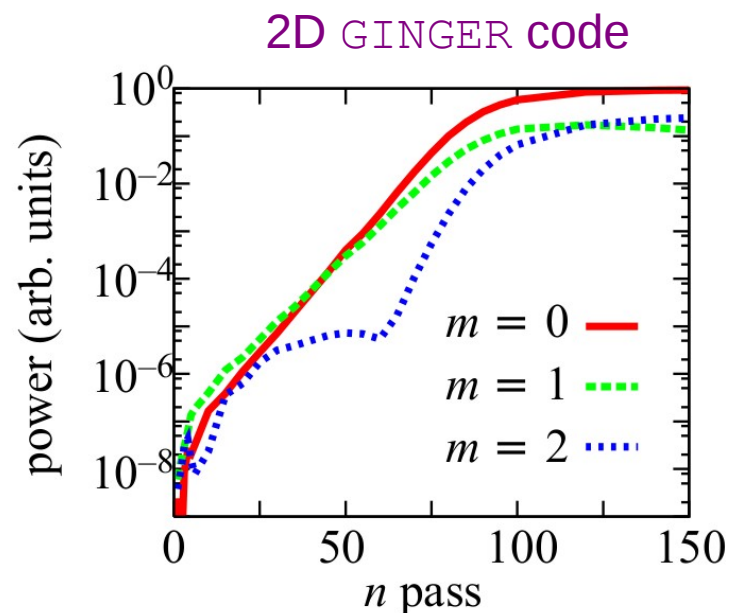
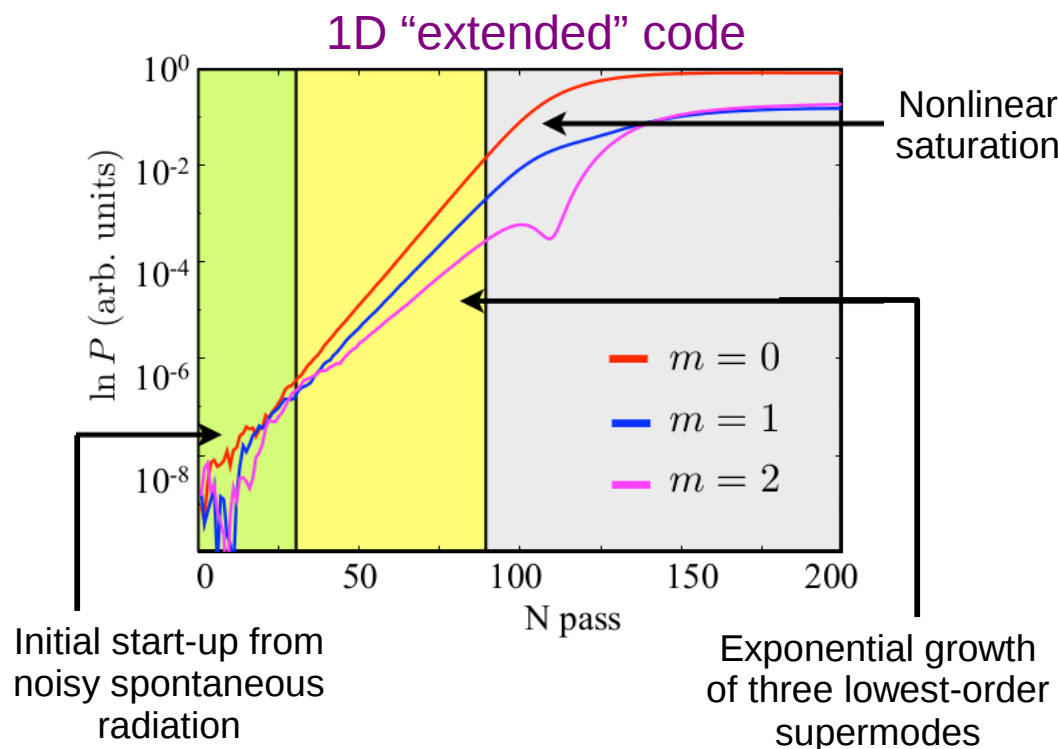
Linear supermodes of the exponential growth regime

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Input radiation brightness

Undulator brightness
(Wigner) function

Electron beam
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Assume Gaussian field in position and angle

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Computed in, e.g., [10]

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Energy spread effect with harmonic (points to σ_γ)

Detuning from resonance (points to x)

Emittance and diffraction effects (points to Σ_x^2 and Σ_ϕ^2)

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Harmonic number (points to $[JJ]_h^2$)

Energy spread effect with harmonic (points to $e^{-2[2\pi N_u(z-s)h\sigma_\gamma/\gamma]^2}$)

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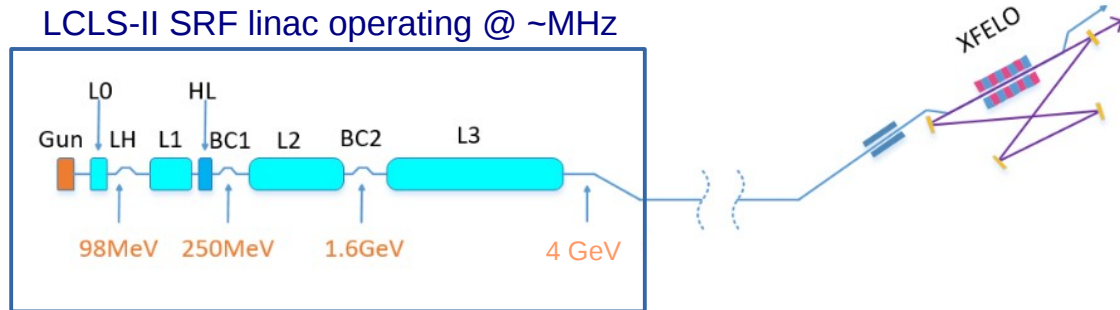
Harmonic XFEL for “low” energy e-beams

- For fixed e-beam parameters, the gain at a harmonic can be larger than that at the fundamental if the energy spread is small enough, $h\sigma_\gamma/\gamma < 1/2\pi N_u$
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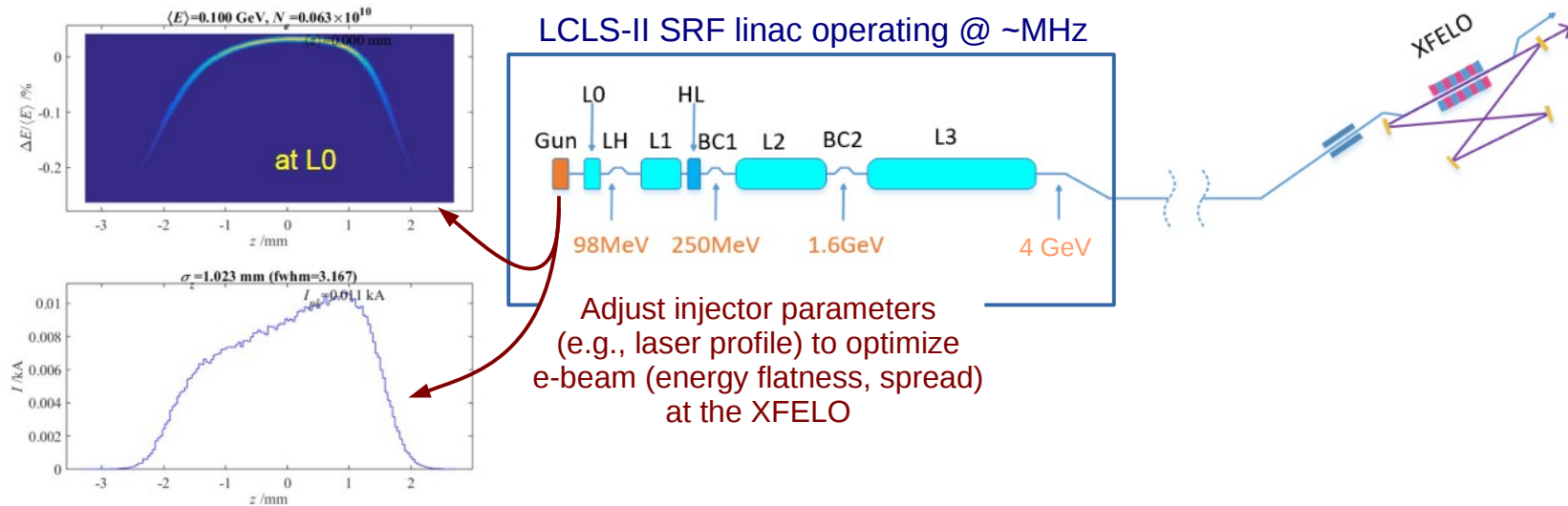


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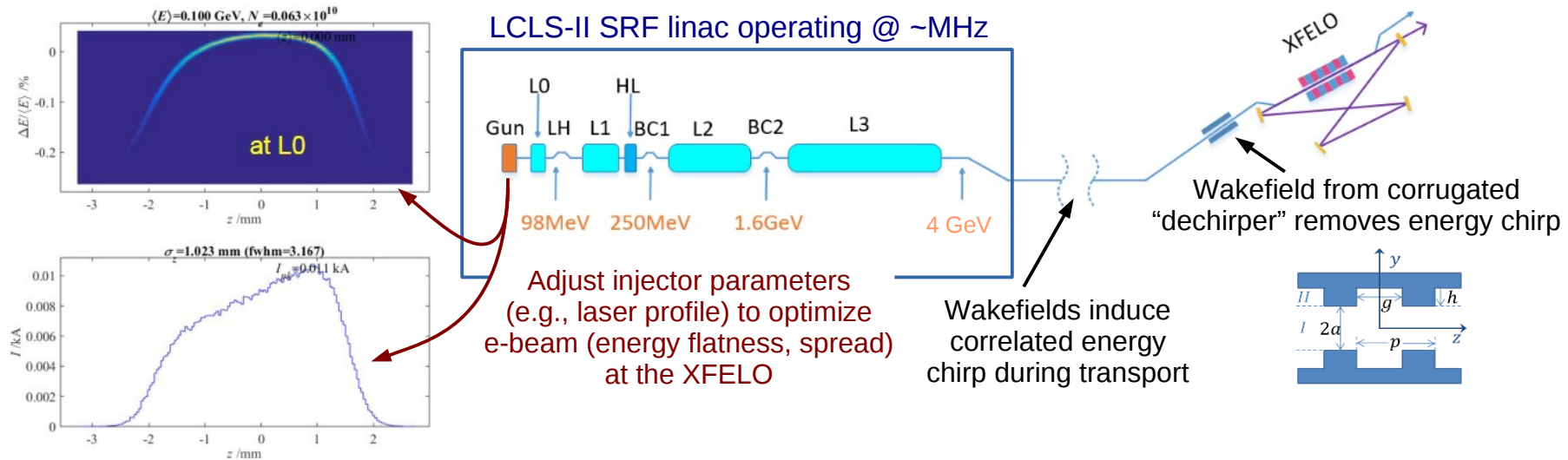


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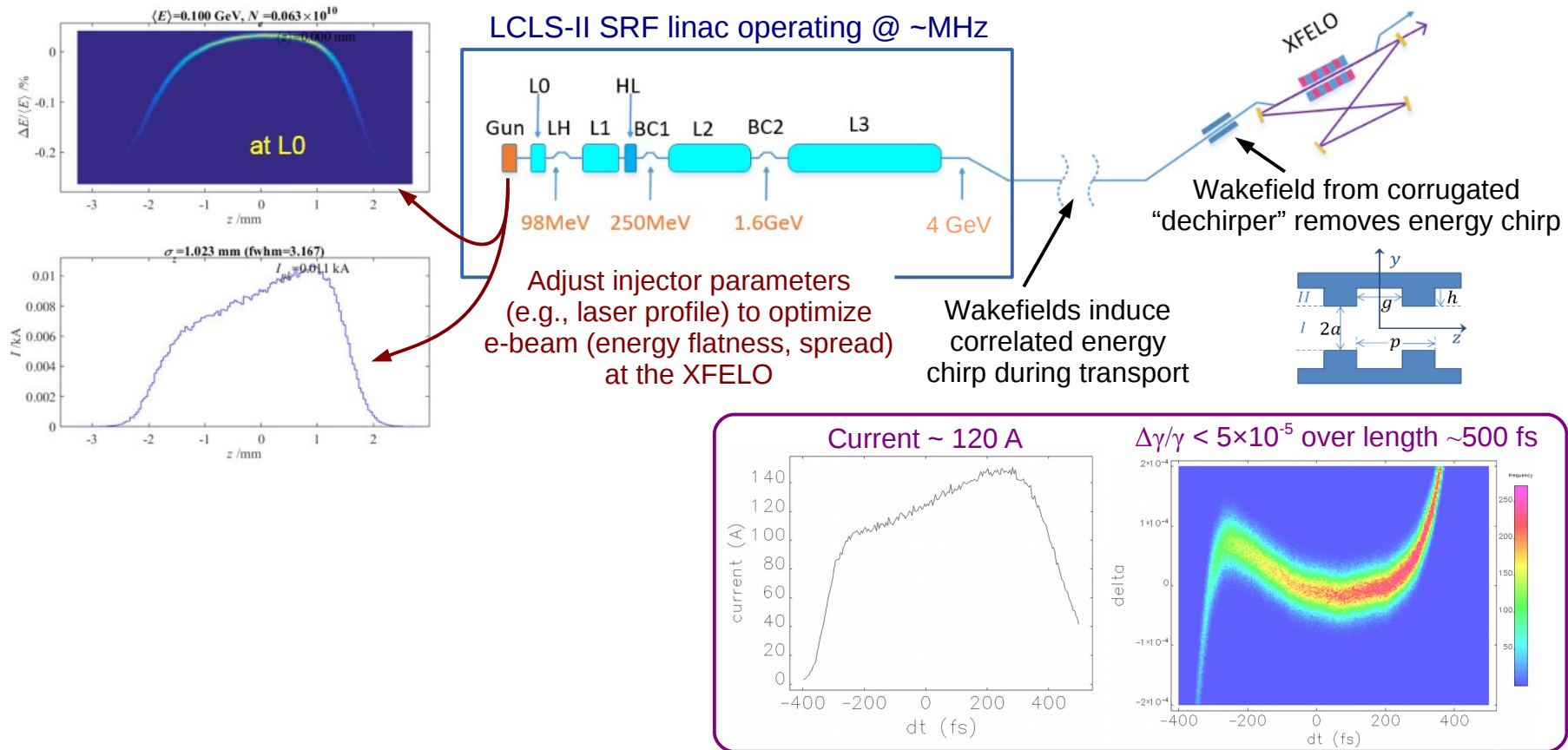


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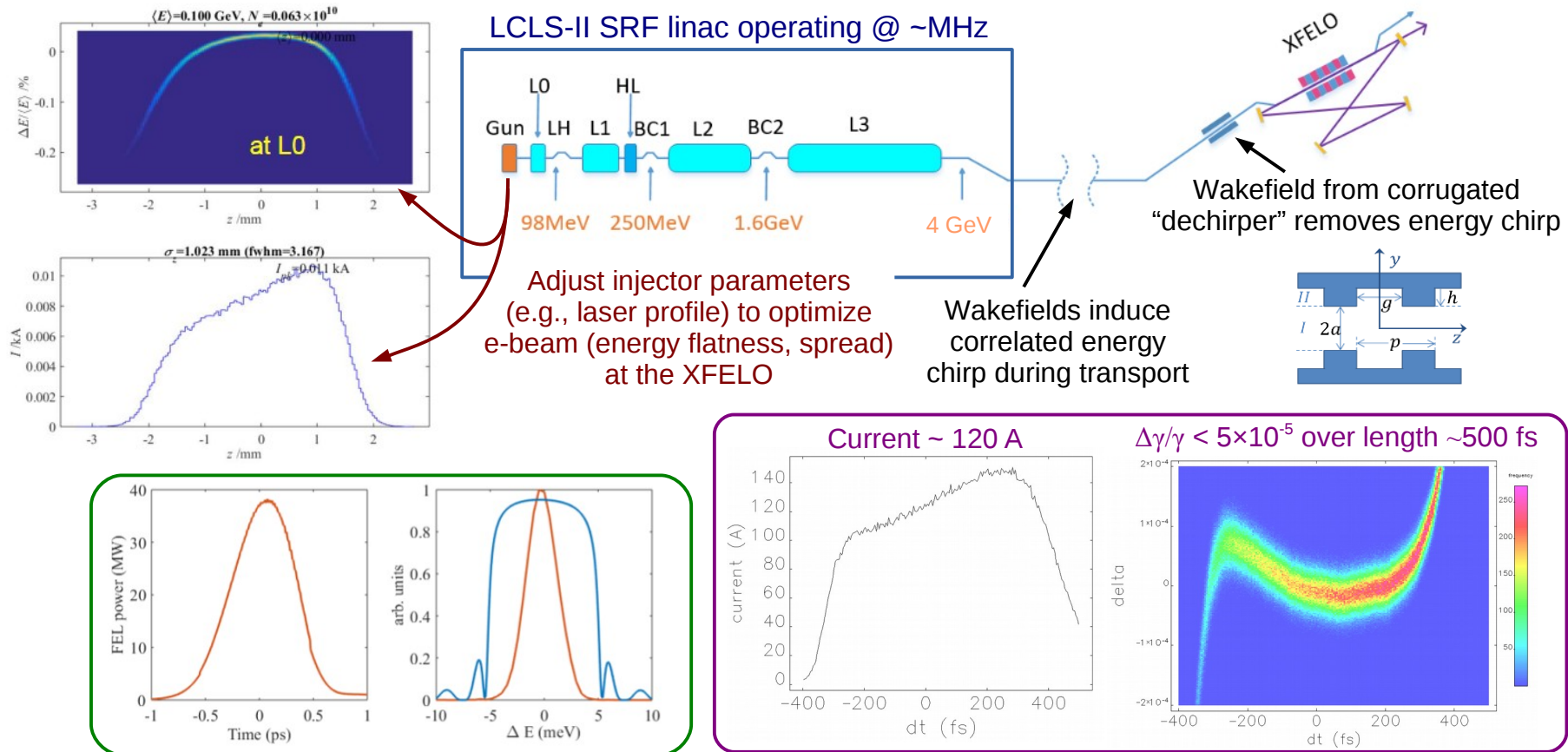


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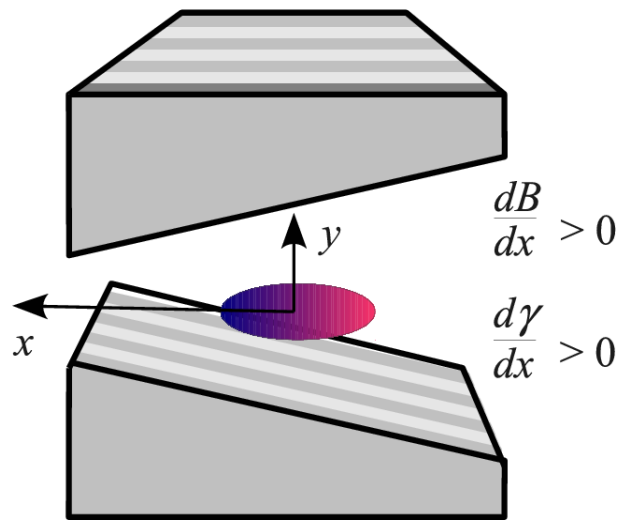
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Reducing energy spread requirement with a transverse gradient undulator

- Gain suffers when the energy spread becomes larger than the FEL bandwidth, $\sigma_\gamma/\gamma > 1/3N_u$
- The energy spread effect can be compensated if we both
 1. Use a transverse gradient undulator (TGU) to vary the FEL resonance with position^[13,14]
 2. Introduce carefully matched dispersion to correlate the energy with position

$$\lambda_1 = \lambda_u \frac{1 + \frac{1}{2}K(x_j)^2}{2\gamma_j^2}$$



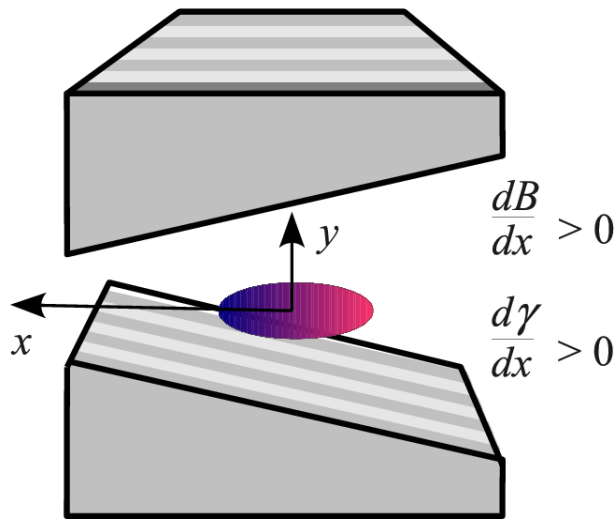
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Introduce dispersion such that $x_j \rightarrow D\eta_j$

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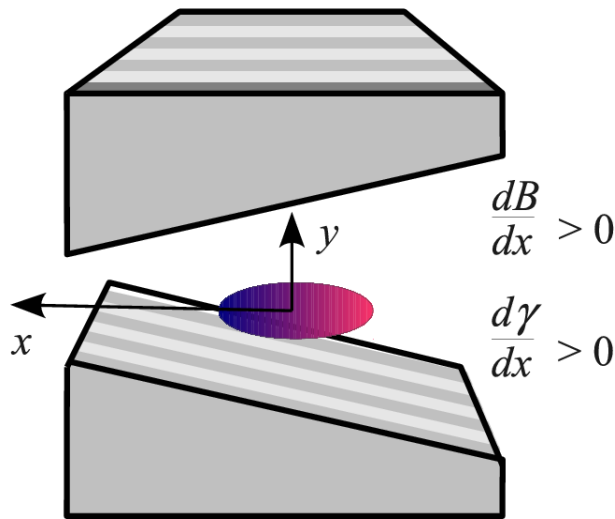
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Energy spread dependence vanishes if we choose

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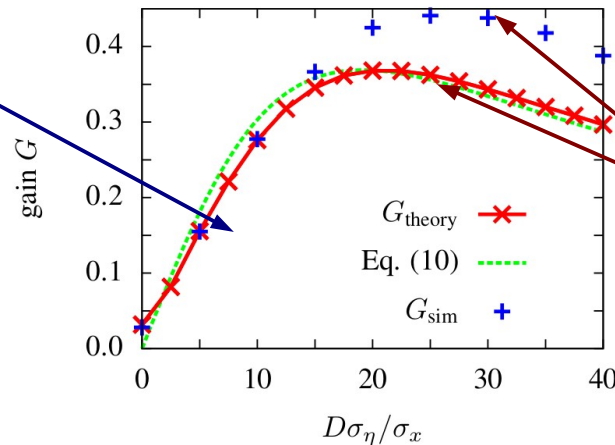
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TGU-enabled XFEL in an “ultimate” storage ring^[15]

Gain is maximized by choosing the dispersion/gradient to balance two competing effects:

Increasing γ - x correlation (dispersion D) reduces the variation in resonance condition due to energy spread, thereby increasing the gain



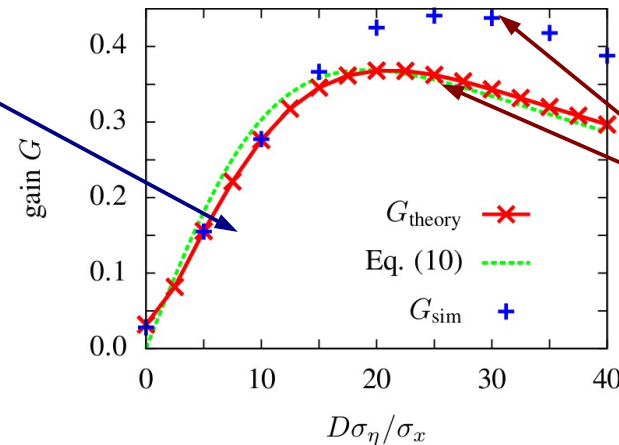
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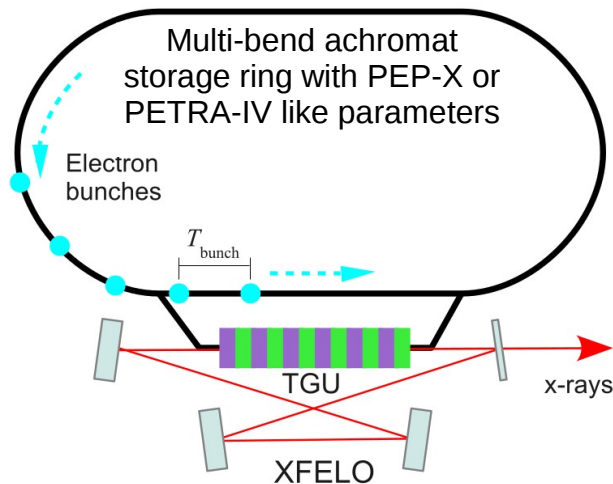
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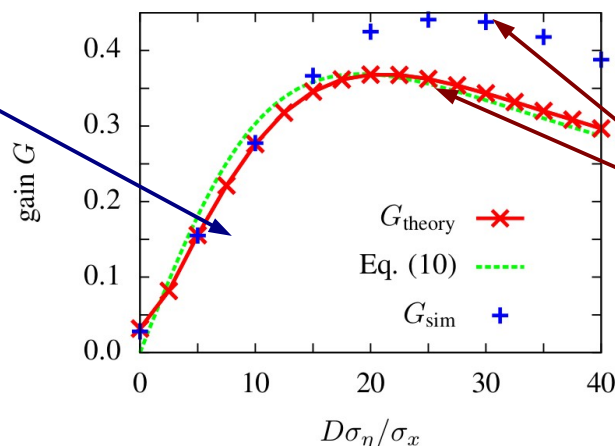


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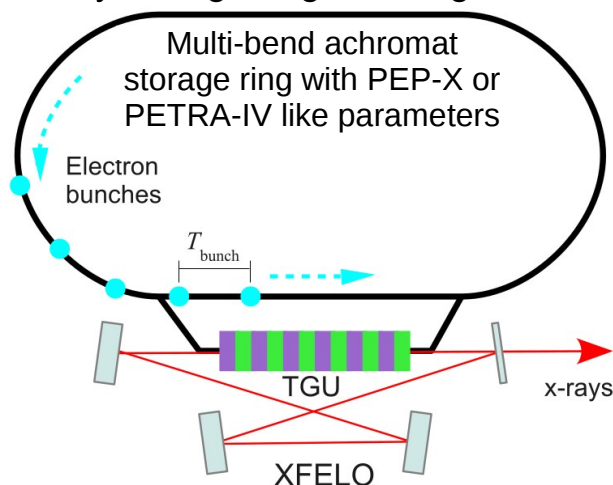
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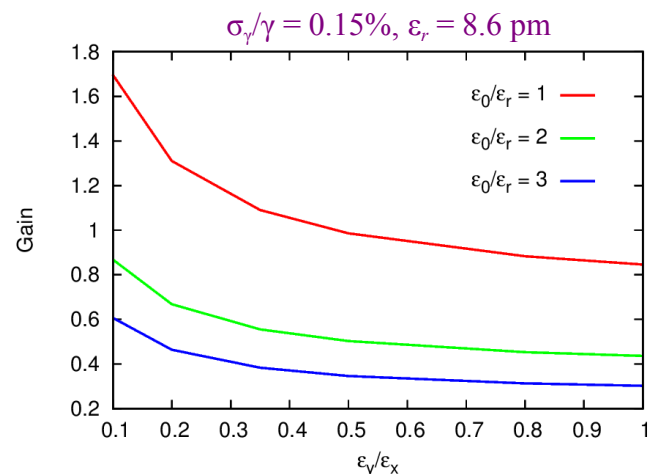


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Decreasing the coupling and dispersing along y yields larger gain



From
Yuan Shen Li

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Tolerances and stability

- Requirements on longitudinal tolerances given by supermode theory

Jitter in arrival time of
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Note that (bunch length) $\lesssim 1/(\text{crystal bandwidth})$

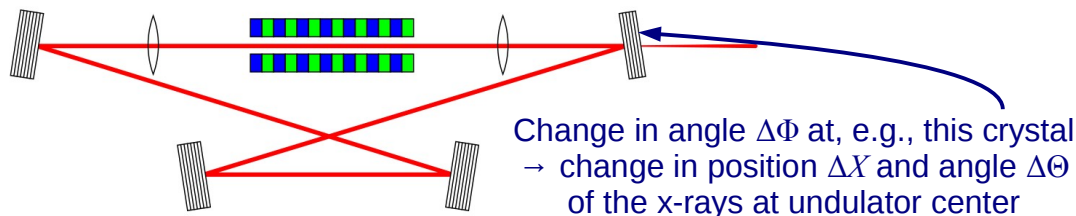
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- Requirements on longitudinal tolerances given by supermode theory

Jitter in arrival time of
electron & photon beams $\ll 1/(\text{crystal bandwidth})$

Note that (bunch length) $\lesssim 1/(\text{crystal bandwidth})$

- Requirements on transverse tolerances of optical components can be found using matrix-type formalism familiar to accelerator physics + requirement to preserve mode overlap



Requiring that
 $\Delta X < [0.1(\text{FEL mode size}) \sim 1 \mu\text{m}]$
implies that $\Delta\Phi < 20 \text{ nrad}$.

Requiring that
 $\Delta\Theta < [0.1(\text{mode divergence}) \sim 0.1 \mu\text{rad}]$
gives a less stringent requirement.

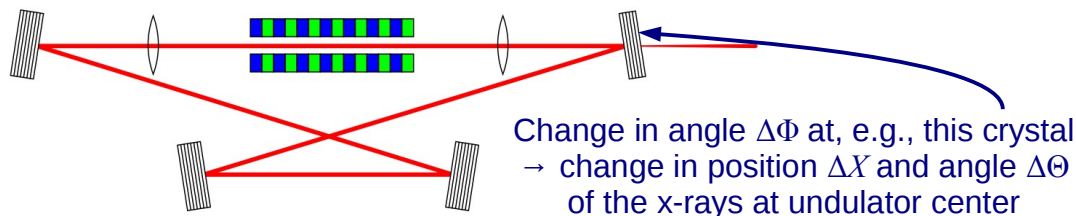
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- Variations over “medium” time-scales require further study
 - Subject of a current grant from DOE/BES

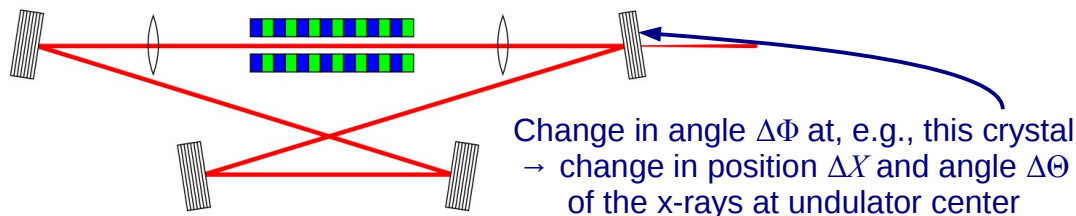
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- Ultimate stability should scale as the (spontaneous power in narrow BW)/(saturation power)

XFEL proof of principle at LCLS-II

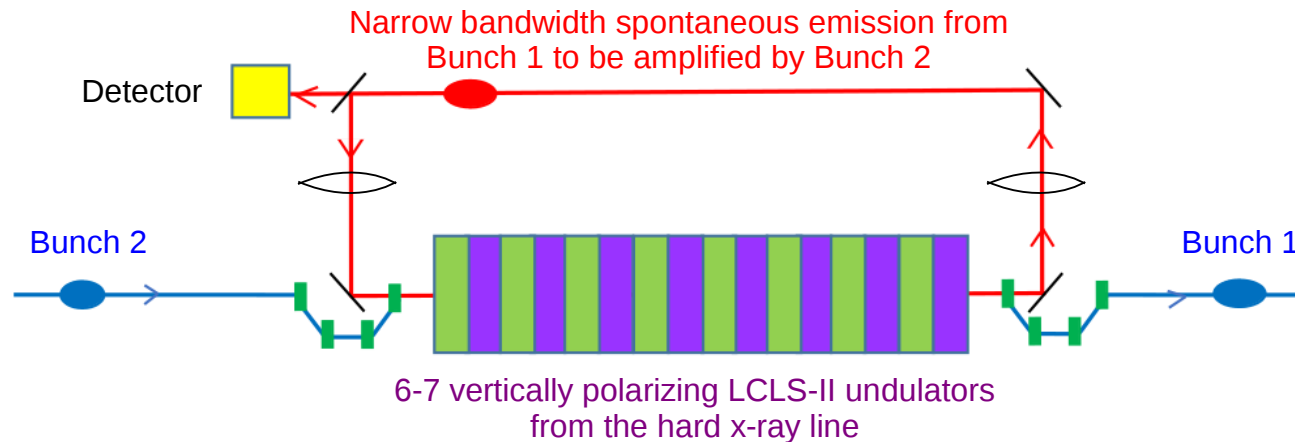
- SLAC has proposed a regenerative amplifier FEL (RAFEL)^[16] at LCLS-II^[17]
 - RAFEL = high-gain oscillator with potentially large losses
 - Route to TW pulses if combined with Q-switching (sudden release of trapped cavity power via, e.g., pulse heating and lattice distortion of Bragg crystal)
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[16] Z. Huang and R.D. Ruth. “Fully coherent x-ray pulses from a regenerative-amplifier free-electron laser,” PRL **96**, 144801 (2006).

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XFELO proof of principle at LCLS-II

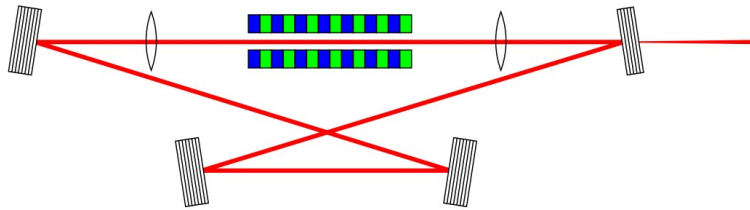
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- XFELO goals are to show sufficient cavity stability to show gain in a two-bunch configuration



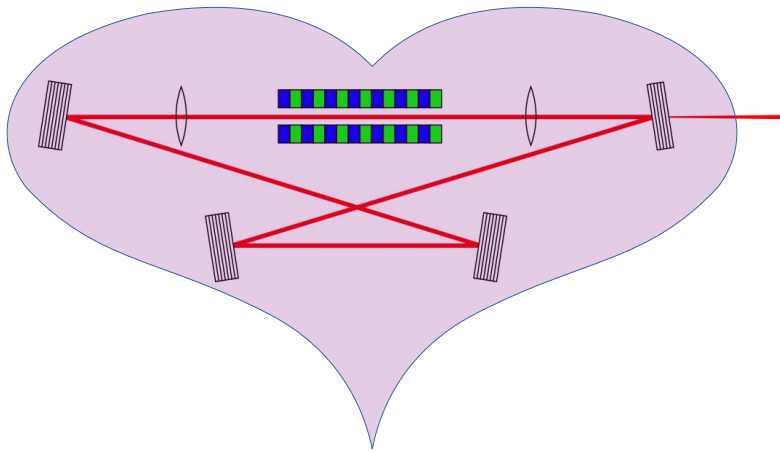
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XFELO as the heart of an “ultimate” x-ray facility

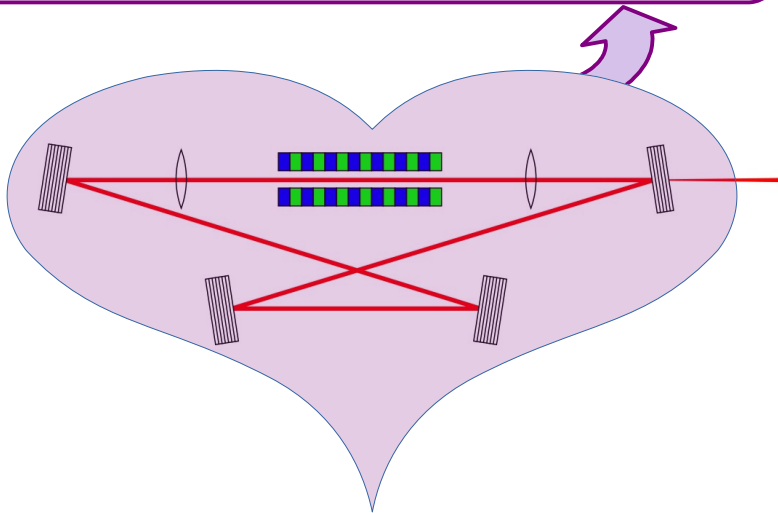
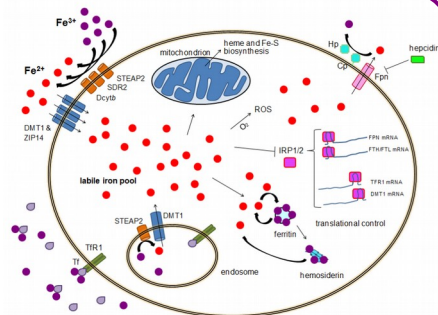


XFELO as the heart of an “ultimate” x-ray facility



XFELO as the heart of an “ultimate” x-ray facility

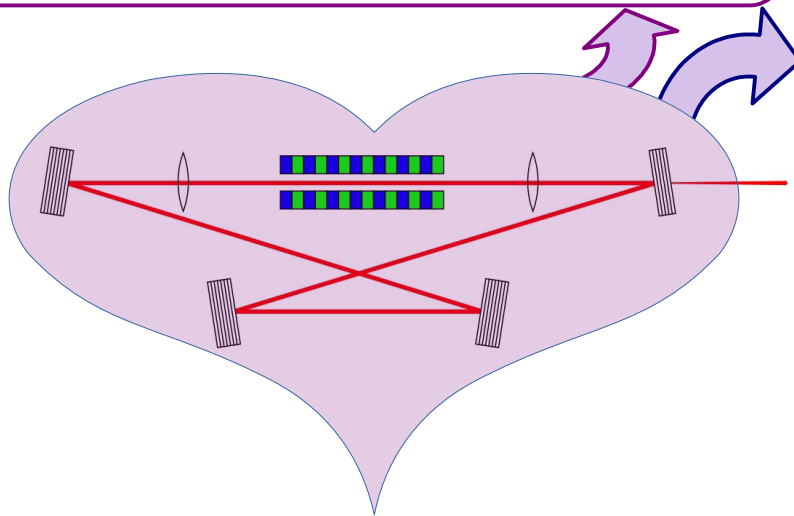
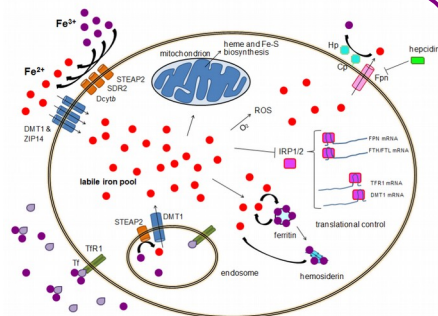
Science relying on fully coherent, 10^9 /pulse with ultrafine (meV) spectral resolution at a MHz rep rate^[18]



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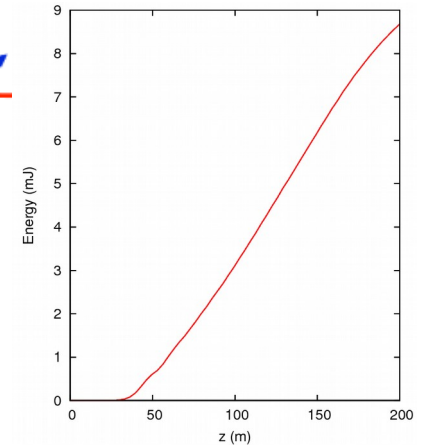
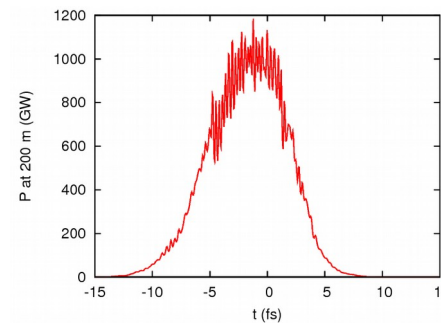
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TW pulses by seeding a tapered high-gain FEL^[19]

LCLS-I amplifier

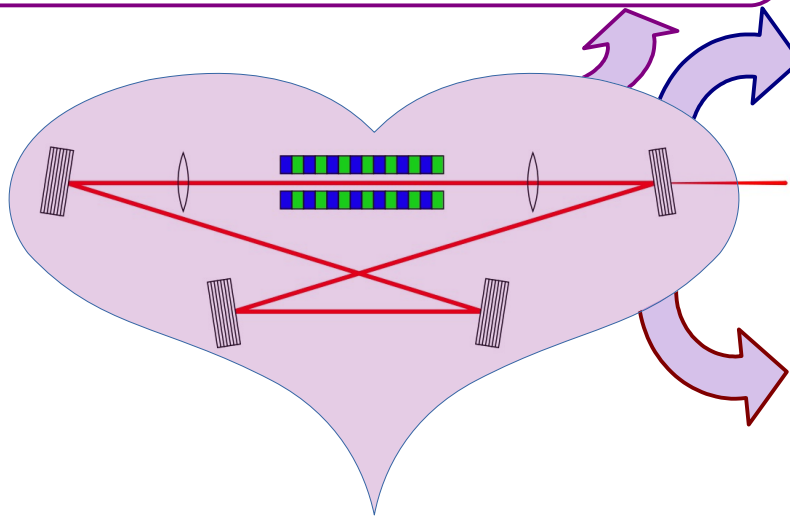
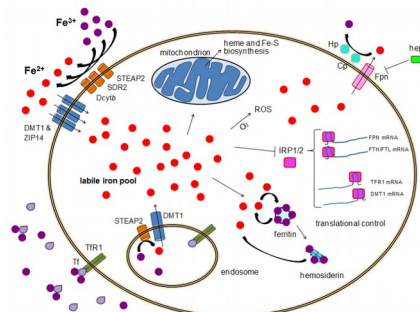


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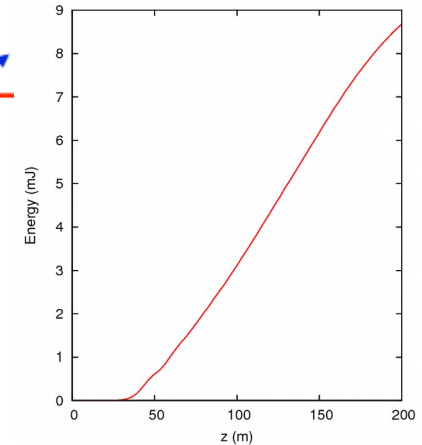
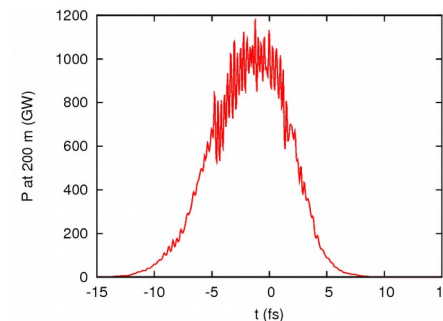
XFEL as the heart of an “ultimate” x-ray facility

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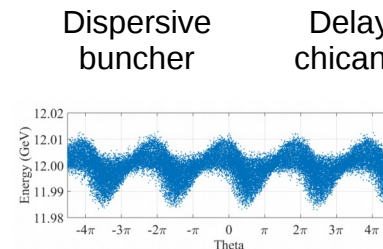


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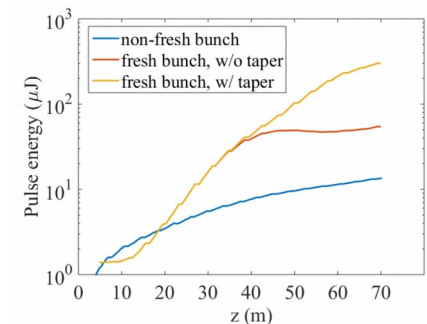
LCLS-I amplifier



> 40 keV photons from HGHG^[20]



Bunching at 4th harmonic
→ 57.6 keV photons



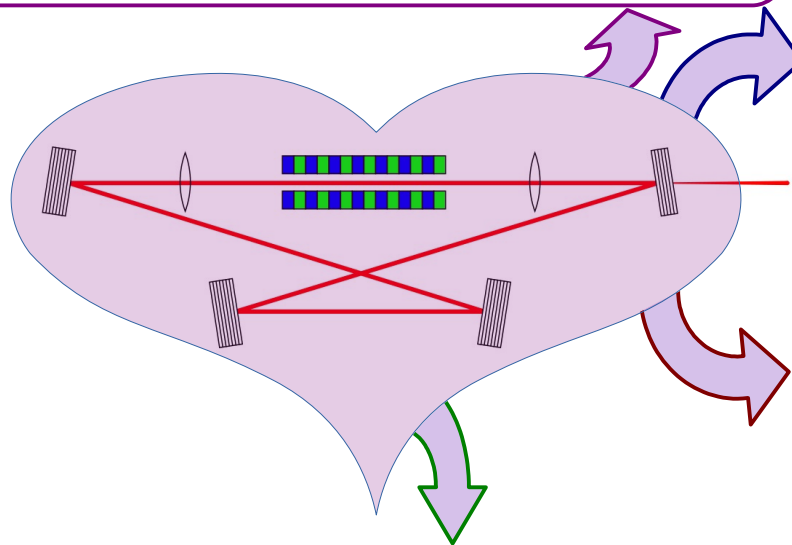
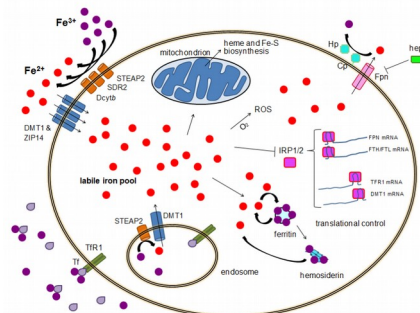
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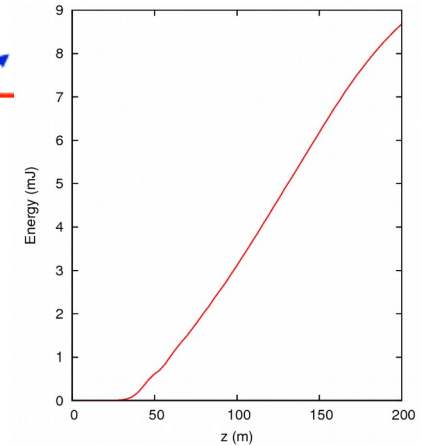
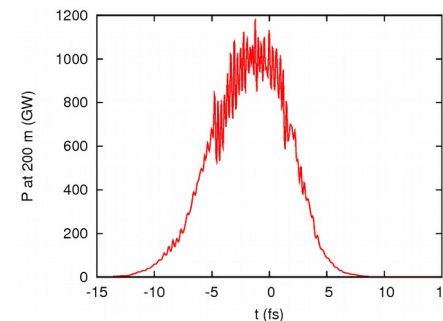
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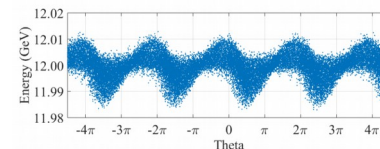
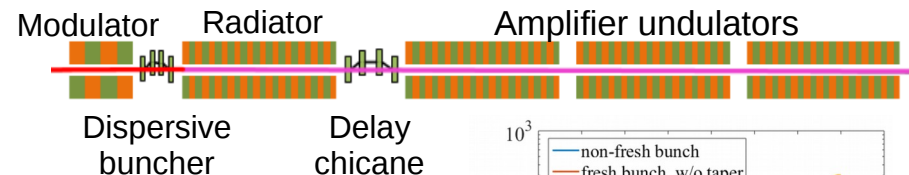
Q-switching for high-power, ultra-narrow bandwidth applications

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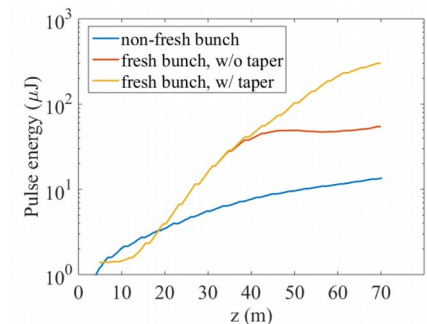
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Acknowledgments

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Thanks for your attention!